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Short communication

Natural convection along vertical wavy surfaces: an experimental study

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Abstract

Natural convective mass transfer coefficients from vertical copper blocks with sinusoidal wavy surfaces of varying amplitude to wavelength ratios (a/λ) have been experimentally obtained. A limiting diffusion current technique based on cathodic reduction of cupric ions was used. The mass transfer performance was observed to decrease with an increasing a/λ ratio. The average Sherwood number can be predicted for a wavy surface with given a/λ by $Sh = 0.915\,Ra^{0.238}(1 + a/\lambda)^{-0.8577}$ within the studied range of Rayleigh numbers.

Keywords: Convection; Amplitude; Concentration

1. Introduction

Natural convection from rough vertical surfaces is of interest in several engineering applications, e.g. heat transfer from electronic circuits and insulation of building walls. Several investigators have studied natural convection from vertical surfaces of different geometries with induced roughness. Sastry et al. [1] reported increase in heat transfer from cylinders wrapped with wires. Jofre and Barron [2] registered up to three times increase in heat transfer from a vertical surface with triangular grooves. Prasolov [3] reported a significant increase for horizontal cylinders with densely spaced pyramids. A marginal increase was reported by Fujii et al. [4] for vertical cylinders with repeated ribs, dispersed protrusions and densely spaced pyramids. However, Heya et al. [5] did not notice any enhancement with pyramids, streak-type, and check type roughness elements on horizontal cylinders. Oosthuizen and Chow [6] studied heat transfer from cylinders with wavy surfaces in various orientations and concluded that the effect of waves on the surface is the least in the vertical direction. Yao and coworkers [7,8] numerically studied heat transfer from a vertical surface with sinusoidal waves under constant surface temperature and constant heat flux conditions. In both situations, heat transfer was inferior to that of a vertical flat plate. Kishinami et al. [9] observed decreased heat transfer from vertical wavy surface formed by alternating semicircular concave and convex surfaces under natural convection. Bhavnani and Bergles [10,11] obtained local free heat transfer coefficients from vertical wavy surfaces and observed decrease in heat transfer with wavy surfaces. Similar conclusions were drawn by Chiu and Chou [12,13] and Kumari et al. [14] for micropolar and non-Newtonian fluids, respectively.

The present work was undertaken to obtain data on natural mass convection from vertical wavy surfaces and to draw general conclusions on the effect of wavy surfaces on natural transport in the present situation. Average mass transfer coefficients from vertical sinusoidal wavy surfaces of different amplitude to wavelength ratios (a/λ) and a flat surface are measured using limiting diffusion current technique (LDCT). The obtained data are discussed in the light of previous literature and correlated to establish an explicit dependency on a/λ ratio.

2. Experimental

The wavy surfaces were machined from solid copper blocks. Nine surfaces of four different a/λ ratios were fabricated. An insulated wire was welded at one end of the machined surfaces for electrical connections. Each sample was painted with an insulating paint such as to expose a

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Nomenclature

a	amplitude of the wavy surface (cm)
A	mass transfer area (cm^2)
C_b	bulk concentration of cupric ions (mol/l)
D	diffusivity of copper ions in acidic copper sulfate solution
F	Faraday's constant
g	acceleration due to gravity (cm/s^2)
i_L	limiting current (mA)
k_L	average mass transfer coefficient (cm/s)
L	vertical distance (cm)
Ra_L	average Rayleigh number, $gL^3(\rho_\infty/\rho_0 - 1)/\nu D$
Sc	Schmidt number, ν/D
Sh_L	average Sherwood number, $k_L L/D$
z	number of electron in the reduction reaction

Greek letters

λ	wavelength of the wavy surface (cm)
ν	kinematic viscosity (cm^2/s)
θ	local inclination angle of the wavy surface
θ_0	maximum value of θ
ρ_0	density of the solution at the surface (g/cm^3)
ρ_∞	density of the bulk solution (g/cm^3)

predetermined area on which mass transfer data were sought. The exposed surfaces were prepared by application of increasing grades of emery papers (100, 400, 600 and 1500 grit size). Subsequently, the active surface was wiped with cotton moistened with acetone to remove any oil/grease. The geometrical details of the prepared surfaces are given in Table 1. The limiting diffusion current technique that has been used to obtain mass transfer coefficients is described elsewhere [15]. Copper reduction from acidic cupric sulfate solution has been chosen as an electrolyte system for its well-defined limiting current plateau. A solution of 0.618 M $CuSO_4$ and 3.09 M H_2SO_4 was prepared in distilled water. The concentrations of copper and acid were measured by idiometric and acid–base titration, respectively.

The prepared wavy surface, held vertically in the electrolyte solution in 1 l glass vessel, functioned as a working electrode. A copper cylinder kept inside the vessel near the wall worked as a counter electrode. A saturated calomel electrode with luggin probe was used for potential measurement. The temperature of the solution was read by a thermometer of 0.1°C accuracy. The working, counter and reference electrodes were connected to a potentiostat (Model 273A EG & G PARC) through an electrometer. The potentiostat was controlled by software (Model 352, EG & G PARC) on an IBM compatible microcomputer. More experimental details are given elsewhere [16].

3. Results and discussion

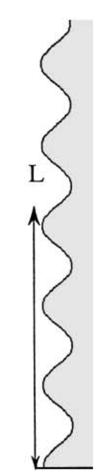
The polarization curves obtained in these experiments show pronounced limiting current plateau facilitating determination of limiting current. For each data point three curves were obtained. The average value of the limiting current and solution temperatures were registered. These values are used to estimate the average mass transfer coefficient (k_L):

$$k_L = \frac{i_L}{zFAC_b} \tag{1}$$

The physicochemical properties of the acidic cupric sulfate solution, which are required to calculate Sh_L and Ra_L, were estimated from a temperature dependent empirical correlation given in [16]. Density of the solution at the surface (ρ_0) is also needed to evaluate Ra_L. Its evaluation necessitates estimation of H_2SO_4 and $CuSO_4$ concentrations at the

Table 1
Geometrical details of each data point[a]

Data	Surface #1 λ = 1.99 cm, 2a = 0.20cm, a/λ=0.050			Surface #2 λ = 1.99 cm, 2a = 0.40 cm, a/λ=0.103			Surface #3 λ = 2.00 cm, 2a = 0.58 cm, a/λ=0.144			Surface #4 λ = 2.01 cm, 2a = 0.79 cm, a/λ=0.198		
Point	L (cm)	P (cm)	A (cm^2)	L (cm)	P (cm)	Ar (cm^2)	L (cm)	P (cm)	A (cm^2)	L (cm)	P (cm)	A (cm^2)
1	0.50	0.51	0.66	0.498	0.55	0.70	0.50	0.59	0.76	0.50	0.67	0.85
2	1.00	1.02	1.32	0.995	1.09	1.41	1.00	1.18	1.53	1.01	1.32	1.70
3	1.49	1.53	1.97	1.493	1.64	2.11	1.50	1.78	2.29	1.51	1.98	2.55
4	1.99	2.04	2.63	1.990	2.18	2.82	2.00	2.37	3.05	2.01	2.64	3.40
5	2.49	2.55	3.29	2.488	2.73	3.52	2.50	2.958	3.82	2.51	3.30	4.25
6	2.99	3.06	3.95	2.985	3.28	4.23	3.00	3.55	4.58	3.02	3.95	5.10
7	3.48	3.57	4.603	3.48	3.82	4.93	3.50	4.14	5.34	3.52	4.61	5.95
8	3.98	4.08	5.261	3.98	4.37	5.64	4.00	4.73	6.10	4.02	5.27	6.80
9	4.48	4.59	5.919	4.48	4.91	6.34	4.50	5.32	6.87	4.52	5.93	7.65



[a] L: vertical length from the leading edge; P: profile length corresponding to L; A: mass transfer area ($L \times P$).

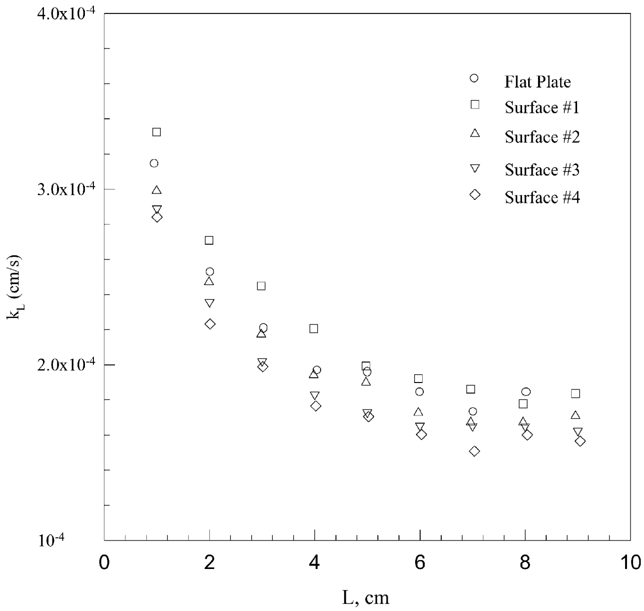


Fig. 1. Average mass transfer coefficients vs. vertical length for various wavy surfaces and flat plate.

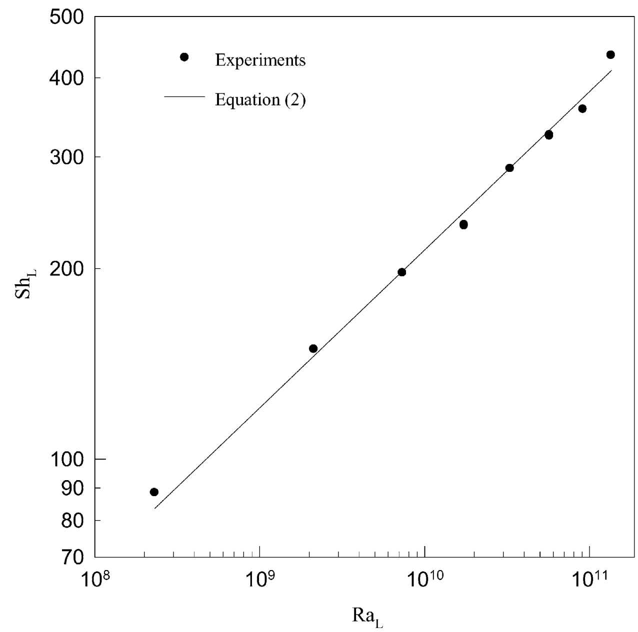


Fig. 2. Average Sherwood numbers vs. Rayleigh numbers for vertical flat plate.

surface. For the mass transfer controlled regime, the surface concentration of $CuSO_4$ can be taken as zero while that of H_2SO_4 is calculated by utilizing principle of the electroneutrality and the correlation given in the reference.

The average mass transfer coefficients for each wavy surface and flat plate are plotted against the vertical distance from the leading edge (L) in Fig. 1. The mass transfer coefficients decrease with increasing length conforming to the laminar boundary layer theory. These curves also indicate that mass transfer is less efficient on surfaces with higher a/λ ratio. However, the dependency of mass transfer coefficients on a/λ cannot be established on the basis of k_L versus x-curves prior to non-dimensionalization of the data because the temperatures were not held constant in all experiments. Therefore, average Sh_L and Ra_L numbers were calculated.

In Fig. 2, Sh_L is plotted against Ra_L number for a vertical flat plate along with the following theoretical equation that was developed through a laminar boundary layer theory [17]:

$$Sh_L = 0.677\,Ra^{0.25}\left[\frac{0.952}{Sc}+1\right]^{-0.25} \quad (2)$$

The curves exhibit excellent agreement of experimental data with the theory of laminar free convection along a vertical flat surface. This establishes the reliability of LDCT and correlations for physicochemical properties of the electrolyte.

Average Sherwood numbers for all wavy surfaces and flat plate are plotted against Ra_L in Fig. 3. It can be observed that the mass transfer performance of the surface with $a/\lambda = 0.05$ is almost the same as that of a flat plate. However, the performance decreases monotonically in the studied range of the Rayleigh numbers. This trend is consistent with the experimental data of Bhavnani and Bergles [10] who obtained heat transfer coefficients for vertical wavy surfaces held in stagnant air using iterferometery. Yao [7] used a transformation method to analyze the current problem mathematically. For $Pr = 1.0$, by plotting $Nu\,Gr^{-0.25}$ against the vertical distance, it was concluded that Nu for sinusoidal wavy surfaces is constantly smaller than that for a corresponding flat plate. When present data are transformed in the same fashion, a similar inference can be drawn.

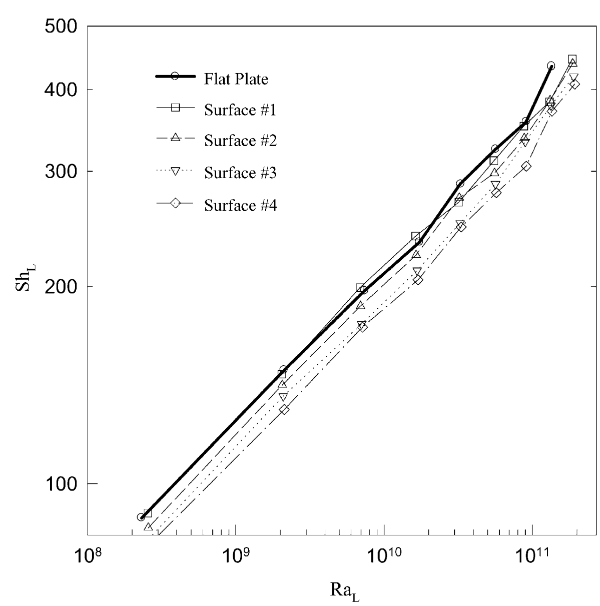


Fig. 3. Variation of average Sherwood number with Rayleigh number for wavy surfaces.

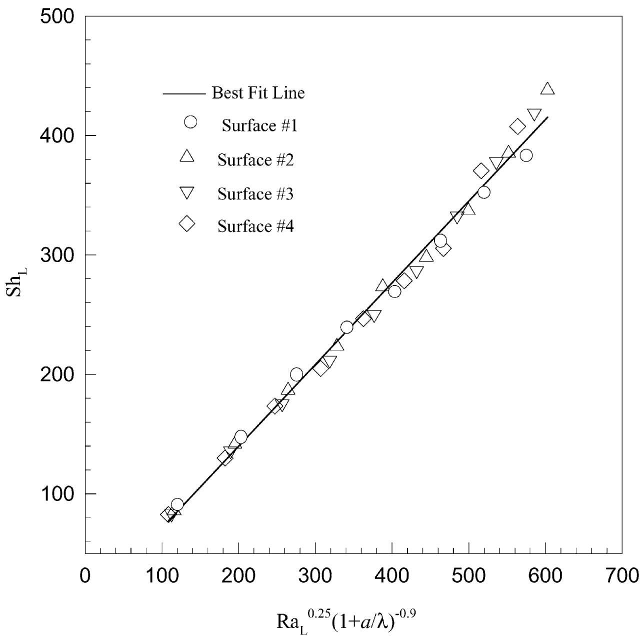


Fig. 4. Sh_L vs. $Ra^{0.25}(1+a/\lambda)^{-0.9}$ plots for all wavy surfaces.

A vertical wavy surface is composed of infinite inclined surfaces of varying inclination angle (θ) whose value changes from $-\theta_0$ to θ_0. Here, θ_0 is a function of a/λ. The inclination angle is zero at the troughs and crests while at nodes its value is $-\theta_0$ or θ_0. When $\cos(\theta)$ equals 1 (i.e. at a trough or a crest), the effect of gravity is the maximum and component of the fluid velocity parallel to the surface is highest. The situation is identical to a flat vertical surface and therefore the local boundary layer thickness and heat/mass transfer coefficients will also be identical. However, at nodes (where $\cos(\theta)$ is minimum) the corresponding values will be the smallest. This will result in lower average heat/mass transfer coefficients. It should also be noted that θ_0 will be large for larger a/λ surface. Therefore, the average heat/mass transfer coefficients for a surface with larger a/λ will be smaller. Similar conclusions can be drawn from Fig. 3.

In an effort to obtain an equation with an explicit dependency on a/λ, the data are plotted as Sh_L versus $Ra_L^{0.25}(1+a/\lambda)^{-0.9}$ in Fig. 4. All the data essentially follow a linear trend. Through a regression analysis the following equation has been deduced with $R^2 = 0.996$:

$$Sh_L = 0.915\,Ra_L^{0.238}\left(1+\frac{a}{\lambda}\right)^{-0.8577} \quad (3)$$

Eq. (3) can now be used to predict average mass transfer coefficients and also average heat transfer coefficients based on an analogy for the range of parameters given in Table 2. When plotted with the experimental data in Fig. 5, this equation shows very good agreement.

Finding the ratio of average Sh_L for a wavy surface and a flat plate can further elucidate the performance of the wavy surfaces. This is done by dividing Eq. (3) by Eq. (2):

$$\frac{Sh_{LW}}{Sh_{LF}} = 1.351\,Ra_L^{-0.0118}\left(1+\frac{a}{\lambda}\right)^{-0.8577}\left(\frac{0.952}{Sc}+1\right)^{0.25} \quad (4)$$

where subscripts W and F represent wavy and flat surfaces, respectively. For the considered range of Ra_L, Eq. (4) is plotted in Fig. 6. The ratio Sh_{LW}/Sh_{LF} decreases sharply with increasing Ra_L in the lower range and asymptotically reaches a constant value. The initial sharp decrease can be attributed to the developing boundary layer. As predicted, the ratio is smaller for a higher a/λ surface at a given value of Ra_L.

Table 2
Range of various parameters in this study

Parameter	Range
Temperature (°C)	21.0–24.3
a/λ	0.050–0.198
Sc	3828–4449
Ra_L	2.3×10^8 to 1.9×10^{11}
Sh_L	82.6–444.9

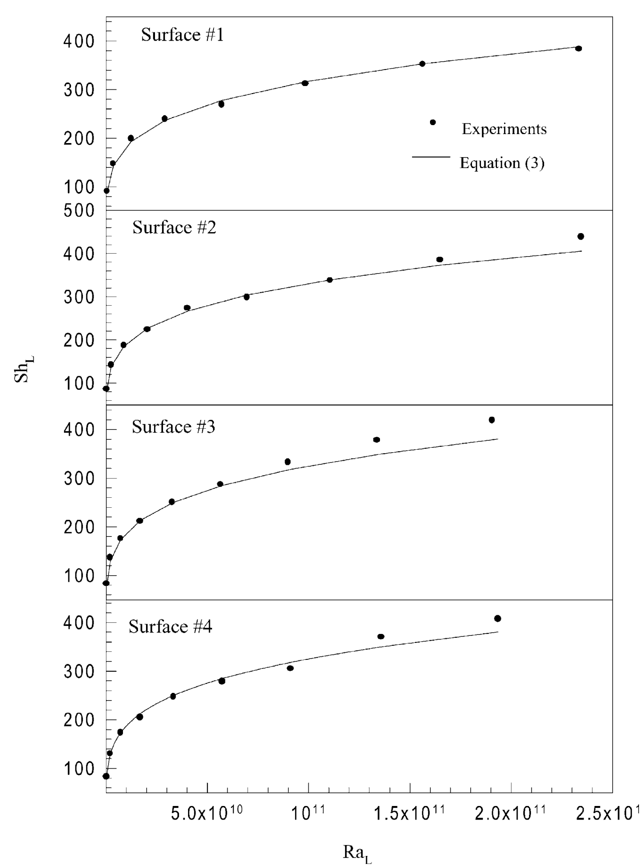


Fig. 5. Comparison of Sherwood numbers predicted by Eq. (3) with experimental data.

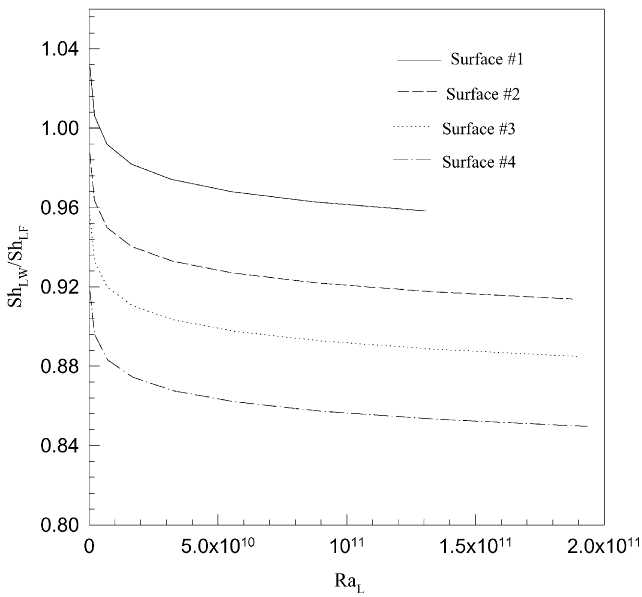


Fig. 6. Variation of Sh_{LW}/Sh_{LF} with Rayleigh numbers.

4. Conclusions

Natural convection mass transfer coefficients from copper blocks with sinusoidal wavy surfaces were obtained using LDCT based on cupric ion reduction in acidic electrolyte. The obtained data are consistent with the previously proposed theory and heat transfer data. A correlation is proposed that allows the prediction of mass transfer performance of a wavy surface with a given a/λ ratio. Based on these data, the following conclusions can be drawn:

1. Mass transfer performance in general deteriorates for wavy surfaces when compared with flat plates.
2. Mass transfer coefficients decrease with increasing a/λ.

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